Solution Sample to HW3

Monday, February 16, 2015
4:57 PM

provided by Junyan

§ 7.4

8. $\int_{-\infty}^{\infty} x e^{-x^2/2}\,dx$

improper because the integral domain is infinite.

$$\int_{-\infty}^{\infty} x e^{-x^2/2}\,dx = \lim_{\substack{a\to-\infty\\ b\to+\infty}} \left[\int_a^0 x e^{-x^2/2}\,dx + \int_0^b x e^{-x^2/2}\,dx\right]$$

$$\underset{du = x\,dx}{\overset{u = x^2/2}{=\!=}} \lim_{\substack{a\to-\infty\\ b\to+\infty}} \left[\int_{a^2/2}^0 e^{-u}\,du + \int_0^{b^2/2} e^{-u}\,du\right]$$

$$= \lim_{\substack{a\to-\infty\\ b\to\infty}} \left(e^{-a^2/2} - 1 + 1 - e^{-b^2/2}\right)$$

$$= \lim_{a\to-\infty} e^{-a^2/2} - \lim_{b\to\infty} e^{-b^2/2}$$

$$= 0 - 0$$

$$= \boxed{0}$$

□

28. $\int_{-\infty}^{1} \frac{3}{1+x^2}\,dx$

When $x \le -1$. $0 \le \frac{3}{1+x^2} \le \frac{3}{x^2}$

$\because \int_{-\infty}^{-1} \frac{3}{x^2}\,dx$ converges $\quad \therefore$ By comparism test, $\int_{-\infty}^{1} \frac{3}{1+x^2}\,dx$ converges

$$\int_{-\infty}^{1} \frac{3}{1+x^2}\,dx = 3\lim_{a\to-\infty} \int_a^1 \frac{1}{1+x^2}\,dx$$

$$= 3\lim_{a\to-\infty} \arctan x \Big|_a^1$$

$$= 3\lim_{a\to-\infty} \left(\frac{\pi}{4} - \arctan a\right)$$

$$= 3\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$$

$$= \boxed{\frac{9}{4}\pi}$$

□

41. $\int_1^{\infty} \frac{dx}{\sqrt{1+x}}$

When $x \ge 1$, $\frac{1}{\sqrt{1+x}} \ge \frac{1}{\sqrt{x+x}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{x}} \ge 0$

$\because \int_1^\infty \frac{1}{\sqrt{x}}\,dx$ diverges by Ex 33.

$\therefore$ $\boxed{\int_1^\infty \frac{dx}{\sqrt{1+x}} \text{ diverges by comparison test.}}$ $\square$

§12.1

20. During International Movie Week, 60 movies are shown. You have time to see 5 movies. How many different plans can you make?

this problem will not be graded

Sl: If you don't care about the order of the movies you watch, then $\boxed{\binom{60}{5}}$

If you do care about the order, then $\boxed{\binom{60}{5} \times 5!}$ $\square$

42. 100 patients wish to enroll in a small study in which patients are divided into 4 groups of 25 patients each. In how many ways can this be done if no patient is to be assigned to more than one group?

Sl: $\boxed{\binom{100}{25} \times \binom{75}{25} \times \binom{50}{25}}$ $\square$

In this last question it is not entirely clear if the order of the groups matter or not.
If it mattered, then the answer would be as above. If it doesn't, then you'd have to divide by 4!.
Both answers will be accepted as correct

Also, extra exercise 3 will be graded, see the solution attached separately.